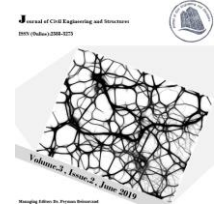




Journal of Civil Engineering and Structures

Journal homepage: www.journalces.com



Original paper

The Effects of Gravity on the Satellites Orbit and Trajectory

Peyman Javadi^{1,*}

¹Master of Geodesy, Lecturer and Director of Geomatics Group of Sama College, Ardabil Branch, Ardabil, Iran.

ARTICLE INFO

Article history:

Received 28 March 2019

Accepted 28 April 2019

Keywords:

Gravity

Orbit

Geoid

Gravity Models

Gravitation Potential

ABSTRACT

The Earth is in the form of a chamfer glob that its diameter at the equator is about 20 km, more than the diameter of the pole. This low flattening (chamfering) plays a disturbance role for the satellites. Regarding to the satellite angular momentum, the orbit behavior, is similar to a gyroscope, and reacts with the motion of an orbital page and it makes of the nodes to some degrees in a day. Apart the secular disturbances of the orbit, non-spherical of the Earth, causes a variety of additional disturbances that effect on the orbital parameters and will have the greatest impact (the most significant effect) on the near-Earth and low-altitude satellites. So far using various methods, the Earth's gravity field and its heterogeneous effect on the direction of the satellite have been studied. The effect of non-spherical nature of the Earth (advent in different geoid models) to the direction of motion and orbit parameters to a satellite sample will be studied in this paper. Using different methods, including ground-based measurements and mapping as well as considering the satellite moving in respective orbits, the gravity (geoid) can be modeled. The accuracy of different geoid models makes some changes in the direction of simulated flying objects. In this study it has been tried to investigate the effect of different gravity models on this simulation with dynamic simulation of a low orbit satellite. First, the relations for modeling of the gravity field have been investigated and then, using different geoid models and (different) Geopotential coefficients, the changing path of these satellites in the orbit (changes in orbital parameters) is analyzed. In the end, to ensure the performance of circuit simulation program, the obtained results of this program will be compared to the findings of the STK software for the sample satellite.

©2019 JCES All rights reserved

1. Introduction

Geoid has the highest accommodation with the irregular and disordered surface of the Earth. But it is a potential coordinate level that almost follows the mean sea level (MSL) (fig1).

In order to have a better understanding of ocean dynamics and interaction of continents, the poles' ice, the sea level studies, and to have a better determination of circuits and height systems in engineering and science, we should increase our knowledge of the Earth's gravity field. And its determination can be possible only through the estimation of global form of the Earth and its inner physics in a specific period by the satellites. The targets are GRACE and CHAMP Satellites. Among the most important global geoids we can name EGM96 (which has a good accuracy for Iran era) and PGM2, A, EIGEN, TEG4, JEM3.

The gravity missions of new satellites CHAMP and GRACE leded us to some major advances in our knowledge about the long-wave length gravity field and therefore to geoid long wave lengths. During these commissions the gravity field information was provided in the form of homogenous and with a global coverage. However, sometimes the error estimations for the global models are too optimistic or (they are) the provider of global average error. However, the performance of a global model for specific and different models will be different. Therefore, the user person or organization of a global gravity model should consider the accuracy of the model, by comparing gravitational field quantities of global model with regional data.

The shape and dimensions of the Earth must be clear. (in the simulation program).At the best way the Earth's shape is estimated as elliptical that forms the basis of geodetic coordinates. (The ellipsoidal base is used in this paper which is based on world

geodetic system of WGS84 model). Here is necessary to present the concept of elliptical base so, we define geoid first. Geoid is defined as the level of the Earth's gravity field and is almost equivalent to the average level of sea. Of course it doesn't mean that the sea level is fixed. Geoid is perpendicular to the gravity vector. Since the mass distribution is not uniform, so geoid has an irregular form of plan metric level (fig1). The main diameter coincides on equator and the small diameter on the rotation axis. In fact, elliptical base which is made by the rotation of an oval, is marked by the size of large and small diameters (or half of them) and the F flattening coefficient.

2. Method and Materials

2.1. Geopotential

Keplerian motion without disturbance it is assumed that the total Earth mass is concentrated in the center of the coordinate system and the gravity law is considered as:

$$\ddot{r} = -\frac{GM_{\oplus}}{r^3}r \quad \ddot{r} = \nabla U. U = GM_{\oplus} \frac{1}{r} \tag{1}$$

The gravity potential can be written (set) according to Legendre polynomial:

$$U = \frac{GM_{\oplus}}{r} \sum_{n=0}^{\infty} \sum_{m=0}^n \frac{R_{\oplus}^n}{r^n} P_{nm}(\sin \phi) (C_{nm} \cos(m\lambda) + S_{nm}(m\lambda)) \tag{2}$$

$$P_{nm}(u) = (1 - u^2)^{m/2} \frac{d^m}{du^m} P_n(u)$$

$$\delta_{0m} = \begin{cases} 1 & m = 0 \\ 0 & m \neq 0 \end{cases}$$

$$C_{nm} = \frac{2 - \delta_{0m}}{M_{\oplus}} \frac{(n-m)!}{(n+m)!} \int_{R_{\oplus}} \frac{S^n}{R_{\oplus}^n} P_{nm}(\sin \phi) \cos(m\lambda) \rho(s) d^3s$$

$$S_{nm} = \frac{2 - \delta_{0m}}{M_{\oplus}} \frac{(n-m)!}{(n+m)!} \int_{R_{\oplus}} \frac{S^n}{R_{\oplus}^n} P_{nm}(\sin \phi) \sin(m\lambda) \rho(s) d^3s \tag{2}$$

Where the coefficients c_{nm} and s_{nm} are called geopotential coefficients and describe the dependency on the Earth internal mass. Geopotential coefficient with $m=0$ are called regional coefficients. They describe a part of potential that is not affiliated to the longitude. Respecting to their definitions all s_{n0} have got zero (0), so we have:

$$J_n = -C_{n0} \tag{3}$$

The even coefficients (J) show the amount of shortness of the polarization axis and the odd coefficients respect the larger amount of the southern hemisphere to the northern one. Due to the complex shape and mass distribution of the Earth, practically J_n coefficients are determined of the information given from satellites motion. These coefficients are called circuit geopotential coefficients.

Other Geopotential coefficients as tesseral are (for $m < n$) and sectorial coefficients (for $m = n$). The model of these coefficients can be seen in fig 5.

Since the Geopotential coefficients c_{nm} and s_{nm} follow the above changes, so usually the normalized coefficients \bar{c}_{nm} and \bar{s}_{nm} are used.

Due to the fact that the inner mass distribution of Earth is unclear, so the Geopotential coefficients can't be determined using equations 2. So far, three general ways have been used to do the observations and measurements in order to improve the gravity models that will be mentioned later.

2.2. Satellite tracking

Since the beginning of spatial flights, manmade satellites' observations allowed scientists to determine the Earth's

gravitational field disturbances through observations of satellites orbital disturbances. The first observation based on photos taken from satellites tracks were done by Baker-Nunn's wide angle telescopes. Development of satellite laser ranging systems in 1965 and their continued development to the accuracy of less than one centimeter increased the knowledge of Earth's gravitational field significantly. Global coverage of the Earth with satellite tracking, made global information retrieval as to long wave length (changes in the gravitational field at large distances for example from equator to the pole) of gravity field possible.

2.3. Measurement of surface gravimetry

The static spring gravity meters measure the local gravitational acceleration to 10^{-3} mGal acceleration (Torge 1991) and therefore, create accurate regional and local information for gravity field. (Short wavelength gravity field changes). Relative gravimetry measures the gravitational differences from one point to another, by diagnosing the inertia action of a typical mass, in response to the change in gravitational acceleration.

Since the gravity measuring has been limited because of the separation of politics and geography, so the airborne gravimeter (with the resolution of 10 to 20 km) or ship-borne have provided the possibility to complete the ground-based measurements with accuracy reduced to 0.1 to 5mGal (Nerem et al 1995). Even according to this data which describe the small scale changes in the Earth's gravity model very well, a special attention is needed for universal over the entire surface of the Earth.

2.4. Altimeter data

Altimeters measure the satellite height above the sea level and they can be used to determine a more accurate height of the mean sea level. Since the resulting height is closely related to the level of potential, altimeter data provide detailed information about the shape of the Earth. Today using a combination satellite tracking the Earth gravimetry and altimetry measurements are used to determine the gravitational field with high accuracy. In calculating the potential gravity, the return equations can be used to calculate Legendre polynomials. Since $p=1$, at the first stage all of the pmm polynomials to the desired rank are computed as the following equations:

$$P_{mm}(u) = (2m - 1)(1 - u^2)^{1/2} P_{m-1,m-1} \tag{4}$$

Where in the above equations u and $(1-u^2)^{1/2}$ are respectively $\sin \phi$ and $\cos \phi$. (ϕ : latitude). According to the obtained results, residual values are resulted from the following equation:

$$P_{m+1,m}(u) = (2m + 1)u P_{mm}(u) \tag{5}$$

And for the $n > m$ values the following recurrence equation is used:

$$P_{nm}(u) = \frac{1}{n-m} \left((2n - 1)u P_{n-1,m}(u) - (n + m - 1)P_{n-2,m}(u) \right) \tag{6}$$

The above equations can be written in a way the Geopotential and its acceleration are a function of the satellite position in Cartesian system (x,y,z). By definition of the following equations:

$$V_{nm} = \left(\frac{R}{r}\right)^{n+1} P_{nm}(\sin \phi) \cdot \cos m\lambda \tag{7}$$

$$W_{nm} = \left(\frac{R}{r}\right)^{n+1} P_{nm}(\sin \phi) \cdot \sin m\lambda$$

In this case the gravitational potential is written as follows:

$$U = \frac{GM}{R} \sum_{n=0}^{\infty} \sum_{m=0}^n (C_{nm} V_{nm} + S_{nm} W_{nm}) \tag{8}$$

V_{nm} And v_{nm} apply in the following recurrence relations:

$$V_{mm} = (2m - 1) \left\{ \frac{xR}{r^2} V_{m-1,m-1} - \frac{yR}{r^2} W_{m-1,m-1} \right\} \quad (9)$$

$$W_{mm} = (2m - 1) \left\{ \frac{xR}{r^2} W_{m-1,m-1} - \frac{yR}{r^2} V_{m-1,m-1} \right\}$$

Also

$$V_{nm} = \left(\frac{2n-1}{n-m} \right) \frac{ZR}{r^2} V_{n-1,m} - \left(\frac{n+m-1}{n-m} \right) \frac{R^2}{r^2} V_{n-2,m} \quad (10)$$

$$W_{nm} = \left(\frac{2n-1}{n-m} \right) \frac{ZR}{r^2} W_{n-1,m} - \left(\frac{n+m-1}{n-m} \right) \frac{R^2}{r^2} W_{n-2,m}$$

If w_{m-1} , v_{m-1} are zero (0), the above equation also will be true for $n=m+1$. Also

$$V_{00} = \frac{R}{r} \quad W_{00} = 0 \quad (11)$$

First with putting $m=0$ in equation (10), the amount of V_n (orbital terms) is obtained that is necessary for V_{nm} calculation. All of the W_n corresponding values are zero, now using the equation (9) and the value V_n , the first terms V_{11} and W_{11} are calculated and in that case all of V_{n1} ($1 \leq n \leq nmax$) will be obtained. The above method is a sustainable method that doesn't allow higher terms to release error calculations during lower terms' calculation.

Acceleration is equal to the u gradient and it can be calculated using V_{nm} and W_{nm} as follows:

$$\ddot{x} = \sum_{n,m} \ddot{x}_{nm} \quad (12)$$

$$\ddot{y} = \sum_{n,m} \ddot{y}_{nm}$$

$$\ddot{z} = \sum_{n,m} \ddot{z}_{nm}$$

Slight accelerations are calculated as follows:

$$\ddot{x}_{nm} \underline{\underline{(m = 0)}} \frac{GM}{R^2} \{-C_{m0}, V_{n,U}\} \quad \underline{\underline{(m > 0)}} \quad (13)$$

$$\frac{GM}{2R^2} \left\{ \begin{array}{l} (-C_{nm} V_{n+1,m+1} - S_{nm} W_{n+1,m+1}) + \\ \frac{(n-m+2)!}{(n-m)!} (-C_{nm} V_{n+1,m-1} + S_{nm} W_{n+1,m-1}) \end{array} \right\}$$

$$\ddot{y}_{nm} \underline{\underline{(m = 0)}} \frac{GM}{R^2} \{-C_{n0}, W_{n,U}\} \quad \underline{\underline{(m > 0)}}$$

$$\frac{GM}{2R^2} \left\{ \begin{array}{l} (-C_{nm} W_{n+1,m+1} - S_{nm} V_{n+1,m+1}) + \\ \frac{(n-m+2)!}{(n-m)!} (-C_{nm} W_{n+1,m-1} + S_{nm} V_{n+1,m-1}) \end{array} \right\}$$

$$\ddot{z}_{nm} = \frac{GM}{R^2} \{(n - m + 1)(-C_{nm} V_{n+1,m} - S_{nm} W_{n+1,m})\}$$

Obviously, if we want to calculate the slight accelerations DF geopotential coefficients to the level C_{nn} and S_{nn} then the $V_{\nu\mu}$ and $W_{\nu\mu}$ terms will be needed to level $n+1$.

The above mentioned relations obtain $j = (\ddot{x}, \ddot{y}, \ddot{z})$ acceleration in the central land device of ECEF and using a transfer matrix, they can be achieved in another device such as inertial device.

3. JEM3 model

A series of coefficients as geopotential coefficients with specific rank and level were used, to model the gravitational field. Now the impact of these coefficients on the gravity model and finally on the satellite motion should be considered. First the JEM3 model with different geopotential coefficients are used for this purpose and the results of the sample satellite with desired specifications are obtained. Comparing this results using circuit emulation application that has been made for this purpose, the impact of gravity with different geopotential coefficients on orbital parameters on a specific period (for example a month), can be examined.

The orbital features of desired satellites used in the simulations are shown in table 1.

Table 1. Orbital features of sampled satellite

Half of the large diameter	6.88km
Eccentricity	0.4
Orbital inclination	55 degree
Longitude of the ascending node	265 degree
Argument perigee	144 degree
Mean deviation	2.5

Table 2. Satellite specifications

Satellite mass	20 kg
Aerodynamic Section	0.25518 m ²
Drag coefficients	1.2
Cross solar	0.28925 m ²
Constant reflectivity	1.5
Satellite mass	20 kg

A three-dimensional view and the way of intern the gravity model into the simulation program can be seen in figs 3, 3, 4. At first, the gravity model of the Earth's ideal mass (which is considered as a spherical body) and the disturbances into the spherical mass of J_2 will be compared. Changes in some orbital parameters for the desired satellites within 24 hours is shown in (7,8,9) charts. Changes in side angle and perpendicular angle to a sample ground station (after 13 hours and 39 minutes that the satellite is seen for the second time), are shown in figs (6, 7) in order to determine these two angles error for the satellite commission for the second time. If we don't consider the J_2 , according to the present simulation, we will have an equals to 6 degrees to the horizon which will be about 10 degrees until the end of the second observation. As well as the horizontal angle, vertical angle changes. If disturbance is not taken into account for the gravity model, the angle changes in the ninth orbital period will be three degrees when the satellite is observed by the device for the second time. If the desired model used in the simulation model considered spherical, for the ninth period when the satellite is visible for the second time, the error of distance will be about 300 km. Some changes in the orbital parameters for J_2 gravity model are shown respectively in figs 10 to 12, as a reference scale for 60 days and nights. Then the effect of different geopotential coefficients on orbital parameters will be specified. These changes are shown in figs 12 to 13, respectively. As it can be seen in fig 13, increasing the geopotential coefficients, the mean deviation starts to increase (with J_2 reference) and the difference won't be significant after J_{20} . (With coefficients and degrees above 20, similar results will be obtained). Perigee argument difference is increased for 20 days but then decreases till the 6th day. Again the difference will not be significant after J_{20} . Changes for slope angle and longitude of the ascending node are decreasing and changes for the slope angle are opposite to the second time and for ascending node longitude are opposite to the third time. Like the other changes of orbital parameters, these changes will not have significant difference for geopotential coefficients above 20. The difference between eccentricities for different geopotential coefficients with J_2 reference are shown in fig (16). At first the difference begins to increase and then after 52 days' decreases. The level of these changes in the greatest increase is about 0.002. By increasing the coefficients after 15 degrees, a significant change can be seen in changes. Now the effect of regarding the nutation and precession on the orbital parameters should be considered. Although the Earth's precession and nutation seem to be minimal and most of the time they can be ignored in calculations, but it

should be noted that the high speed satellite around the Earth for a long term, will make us some unpredicted changes. The effect of moon and sun and other planets on the Earth motion is one of the reasons for these changes that causes the precession and nutation. As seen in figs 21 to 23, changes in the orbital parameters within 60 days regarding to JGM₃ model seem insignificant for both states of the presence and absence of nutation and precession. But is should be noted that these changes show themselves very well after the satellites with long life and they have a negative effect on calculations and predictions.

As it can be seen the precession and nutation have the greatest impact on the mean deviation parameters and the orbit perigee argument, so that after 60 days regardless of precession and nutation of mean deviation they can estimate 9% of errors in calculations. Other models: As mentioned, there are different models of gravity field. Each one of these models affects the satellite motion in a specific way. The effects of five different models (PGM200A, ELGEN, EGM96, TEG4, JGM3) on the orbital parameters of sample satellite have been considered and they are shown in figs 24 to 29 as samples.

4. Compare the results of circuit simulator with STK5

To ensure the performance of circuit simulator program, the results of this program are compared to the results of STK program for desired satellite. To do this, the EGM96 model with J₂ and J(70,70) are used, because the model with this coefficient is (exists) in both programs. The maximum coefficients used in different gravity model are presented in STK guide and it has been found that the maximum rank and level that is supported by EGM96 is (equals to) 70. The calculation results and comparing simulations to the orbital parameters show (in the first 20 days) that there is no significant difference between the results of simulation model with STK in the first 20 days, but after 20 days the results were different from each other and the graphs kept aloof. This difference could be due to computational method, integration way, and the essential error in the two original programs.

5. Summary

According to what was said we define that chamfering of the Earth makes some actions on a satellite near it. Precession: If the orbital plane is inclined and makes an angle with the equator, then it rotates around the poles' axis of the Earth and reverse the satellite circulation (Ω changes). Circuit rotation: If the satellite orbit is elliptical, its large diameter will rotate in the orbital plane. The direction of this rotation is dependent on the slope of the orbital plane. If $i < 63.4$, this rotation is in the same direction with the satellite, and if $i > 63.4$, the rotation reverses the satellite direction (W changes). Increasing the level and rank of geopotential coefficients, the accuracy of gravity model increases. Satellite motion simulation using different methods at low altitudes will have more changes and so there is a great difference of different models in response to high altitude. Increasing in time simulation, the difference of orbital parameters will increase for different models. Also decreasing the satellite height (altitude), the gravitation effect will increase and non-spherical of the Earth makes the satellite motion more turbulence. Circuit simulation software practically showed that when the high ranked gravity models are considered, the results for the orbital parameters are very close to each other and we can ensure of proper function of each of them in high ranks and levels. The orbital parameters are very close to each other and we can ensure of the proper function of each of them in high ranks and levels.

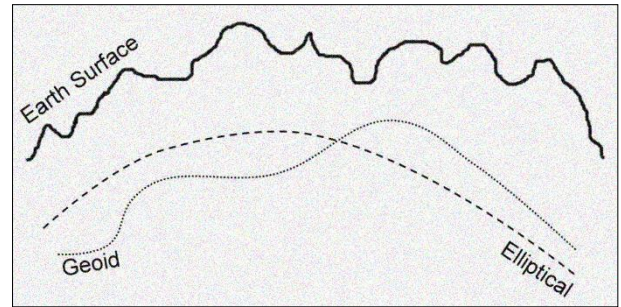


Fig 1. The difference between geoid and elliptical

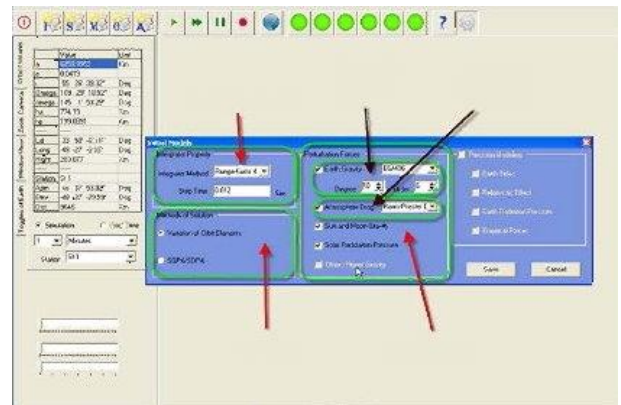


Fig 2. The window of computational and turbulence models

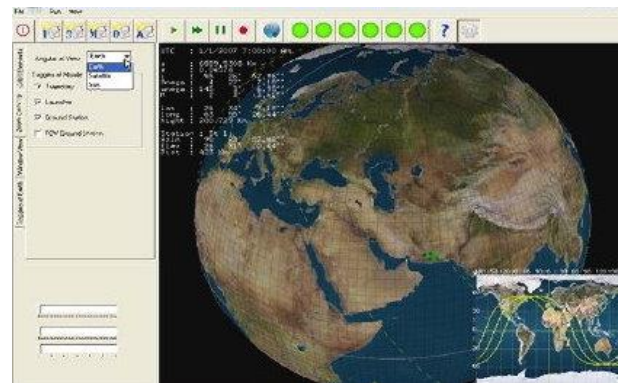


Fig 3. A three dimensional view of satellite motion simulation

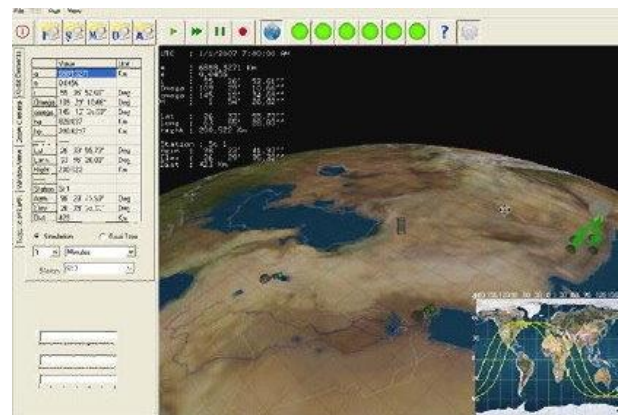


Fig 4. A three dimensional view of satellite motion simulation

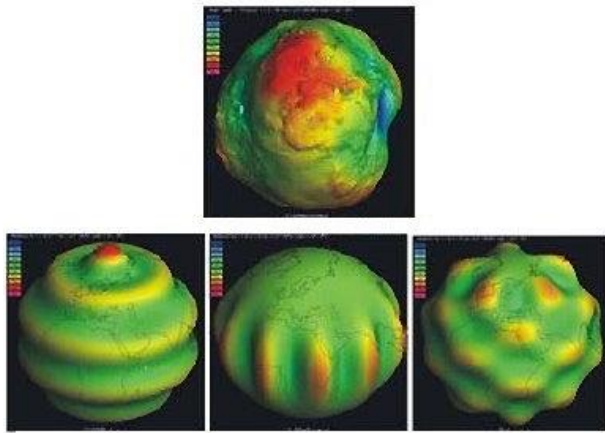


Fig 5. Sectional and orbital spherical harmonic model (respectively from left to right)

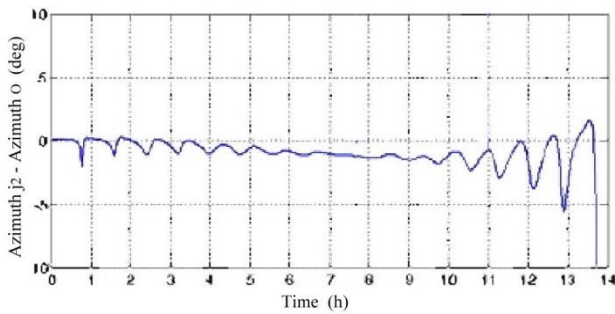


Fig 6. Changing in the angle of the horizon for two spherical gravities and J_2 turbulence of the Earth

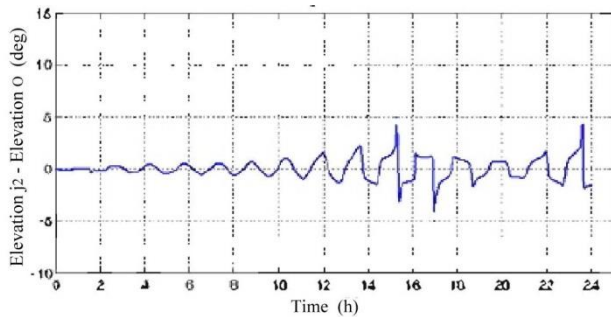


Fig 7- Comparing the changes of perpendicular angle with time for 2 spherical gravities and J_2 turbulence of the Earth.

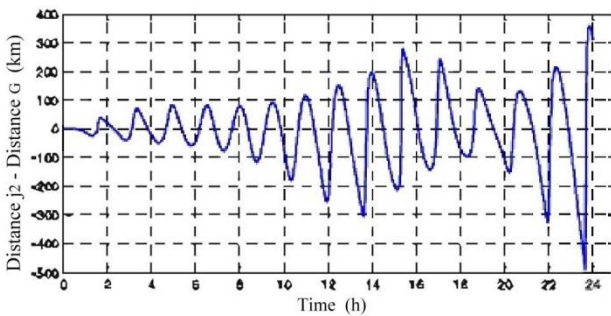


Fig8. Comparing interval changes with special gravities and J_2 turbulence of the Earth

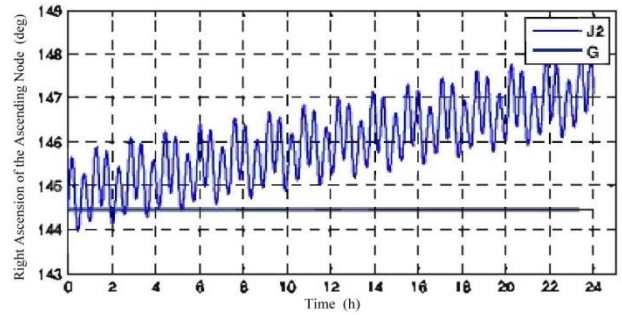


Fig 9. Comparing the changes of ascending node longitude with time for 2 spherical gravities and J_2 turbulence of the Earth

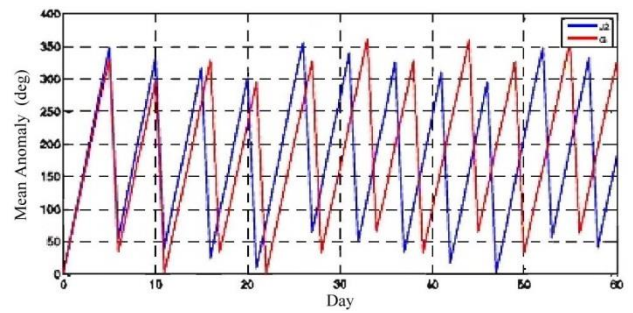


Fig 10. Changes of mean deviation for G, J_2 and their differences within 60 days

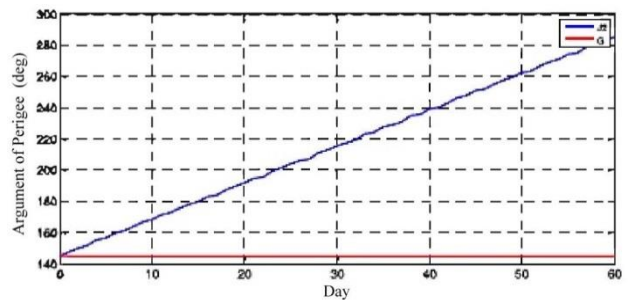


Fig 11. Changes of perigee argument for G, J_2 and their differences within 60 days

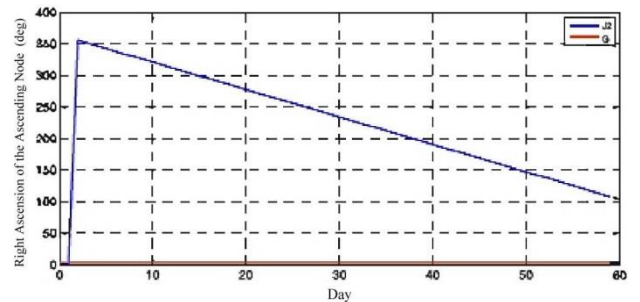


Fig 12. Changes of ascending point longitude for G, J_2 and their differences within 60 days

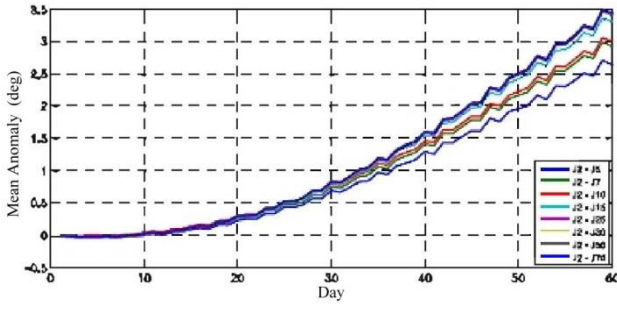


Fig 13. The effect of geopotential coefficients on mean deviation of sample satellite (with J_2 reference)

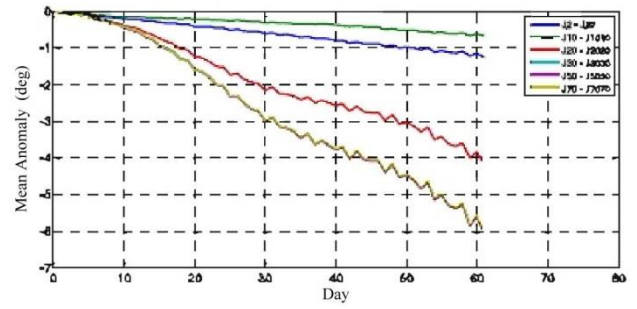


Fig 17. Comparison of mean deviation due to regional and sectional geopotential coefficients.

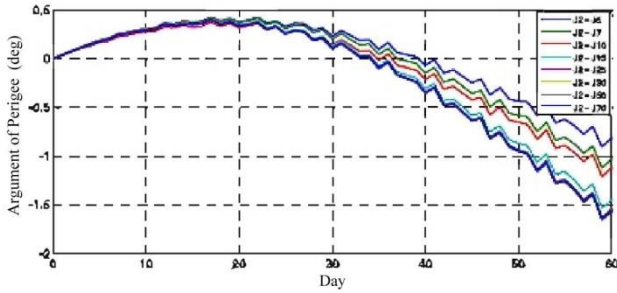


Fig 14. The effect of different geopotential coefficients on the perigee argument of sample satellite (with J_2 reference)

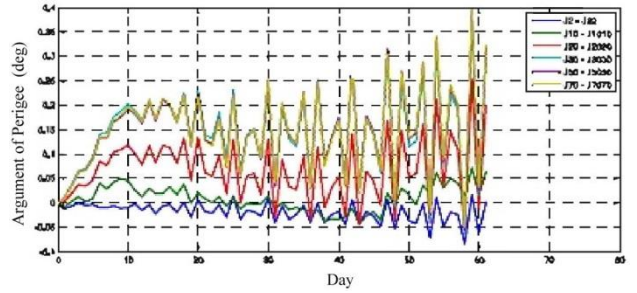


Fig 18. Compare the perigee argument due to regional and sectional geopotential coefficients

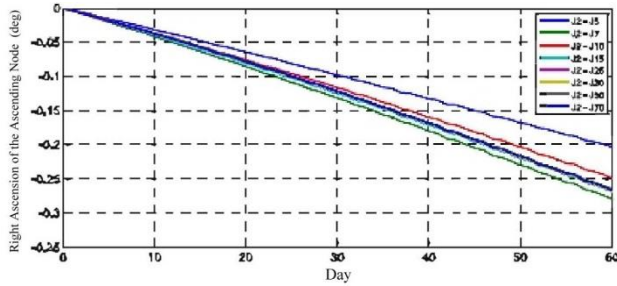


Fig 15. The effect of different geopotential coefficients on the ascending node longitude of sample satellite

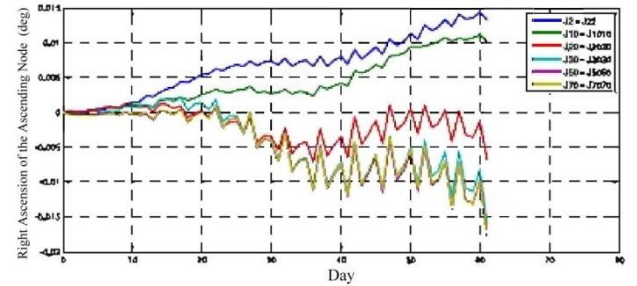


Fig 19. Compare the ascending node longitude due to regional and sectional geopotential coefficients

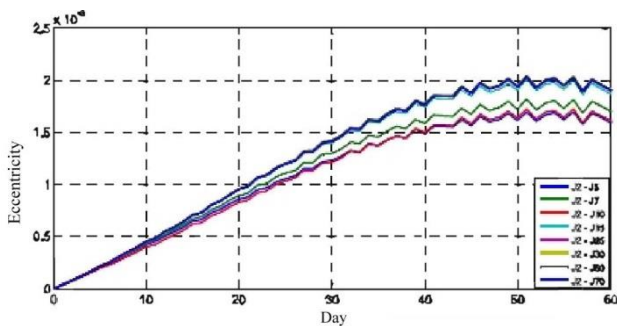


Fig 16. The effect of different geopotential coefficients on the orbital plane slope angle of the sample satellite

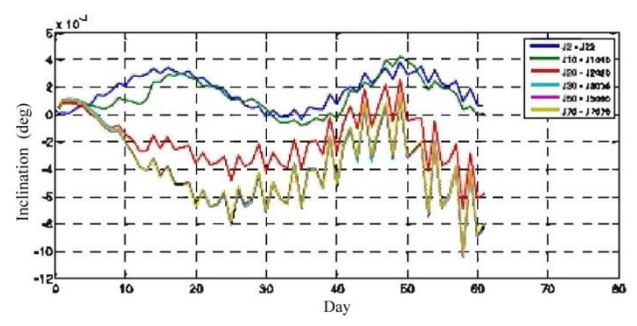


Fig 20. Compare inclination angle due to regional and sectional geopotential coefficients

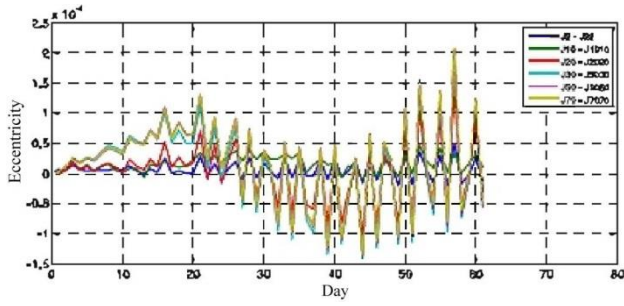


Fig 21. Compare the centrality due to regional and sectional geopotential coefficients

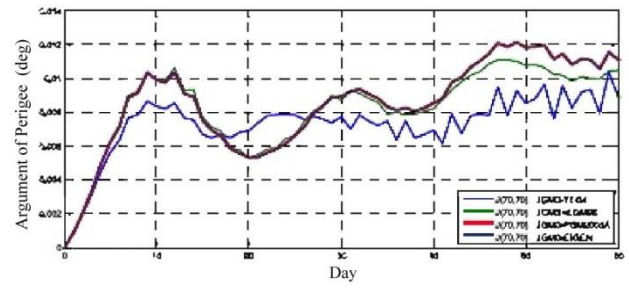


Fig 25. Changes of perigee argument for different models into JGM3 model with geopotential coefficients of 70 rank

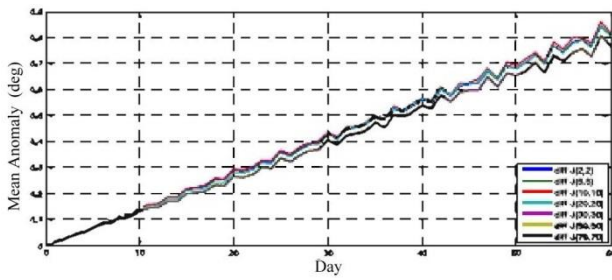


Fig 22. The mean deviation difference for both the presence and absence of precession and nutation

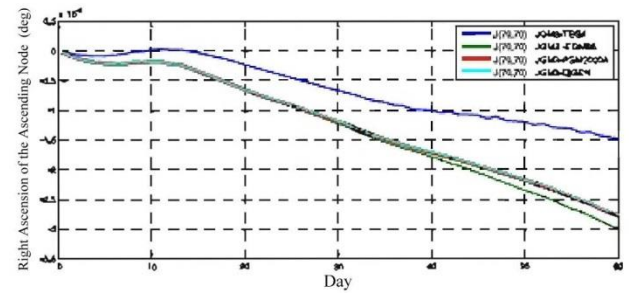


Fig 26. Changes of ascending node longitude difference for different models into JGM3 model with geopotential coefficients of 70 rank

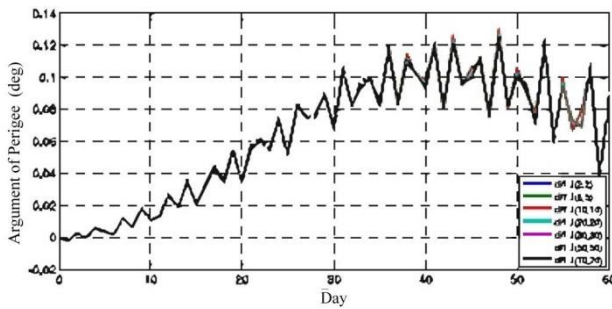


Fig 23. Perigee argument for both the presence and absence of precession and nutation

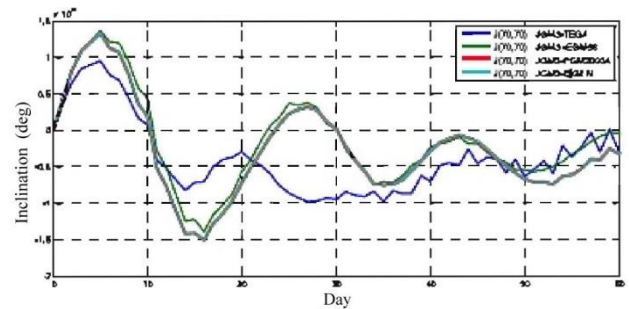


Fig 27. Changes in orbital inclination angle for different models into JGM3 model with geopotential coefficients of 70 rank

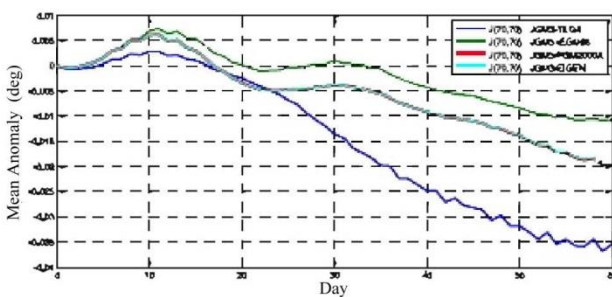


Fig 24. The mean deviation changes for different models into JGM3 model with geopotential coefficient of 70 rank

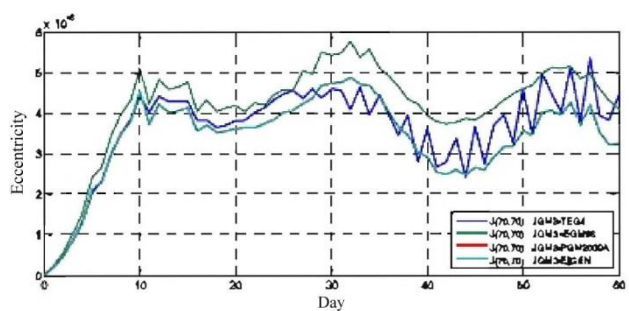


Fig 28. Changes of difference eccentricity for different models into JGM3 model with geopotential coefficients of 70 rank

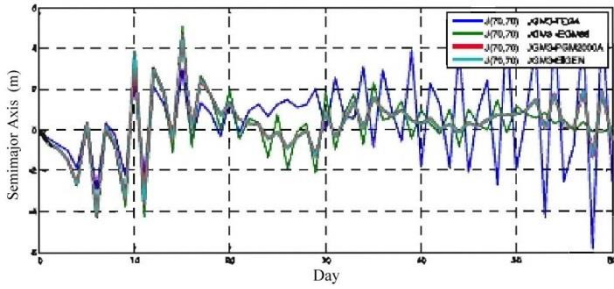


Fig 29. Changes of half of the main diameter difference for different models into JGM3 model with geopotential coefficients of 70 rank

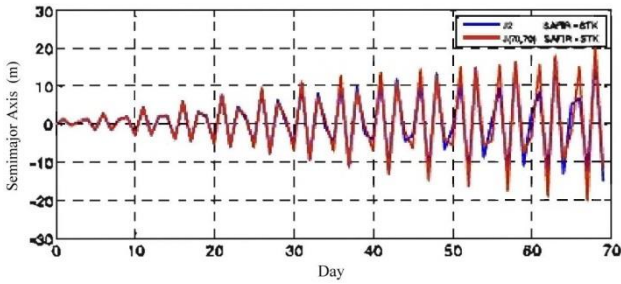


Fig 30. Changes of half of the main diameter with time for two circuit simulation programs and STK and for J_2, J(70,70)

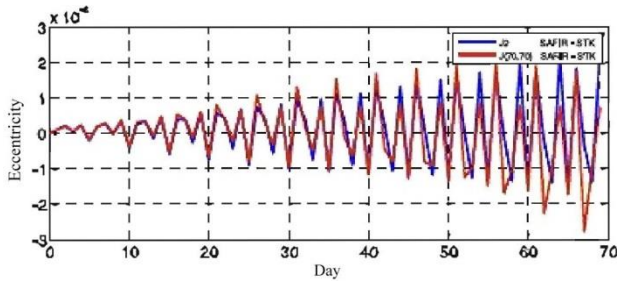


Fig 31. Changes in the eccentricity difference for two circuit simulation programs and STK and for J_2, J(70,70)

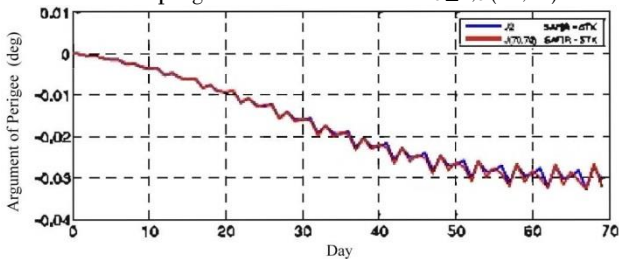


Fig 32. Changes of perigee argument difference with time for two circuit simulation programs and STK and for J_2, J(70,70)

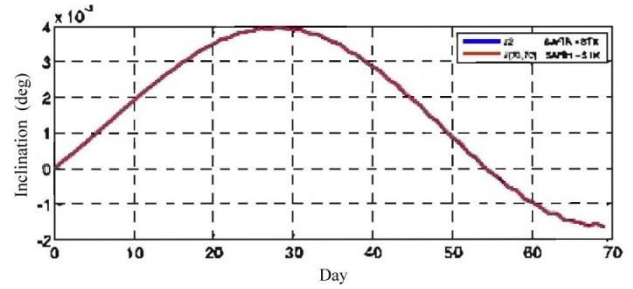


Fig 33. Inclination difference changes with time for two circuit simulation programs and STK and for J_2, J(70,70)

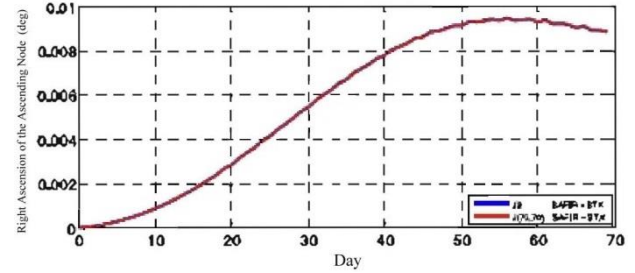


Fig 34. Ascending difference changes from the right side of right point with time for two circuit simulation programs and STK and for J_2, J(70,70)

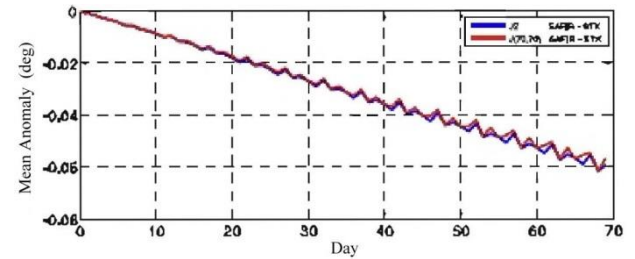


Fig 35. Changes of mean deviation difference with time for two sample programs and STK and for J_2, J(70,70)

References

[1] P. Ditmar, V. Kuznetsov, A. A. van Eck van der Sluijs, E. Schrama, and R. Klees. 2004. "Computation of the Earth's Gravity Field Model from Satellite Accelerations". *Journal of Geodesy*, 78, 12-33.

[2] P. Ditmar, V. Kuznetsov, A. A. van Eck van der Sluijs, E. Schrama, and R. Klees. 2004. "a Model of the Earth's Gravity Field Computed from Accelerations of the CHAMP Satellite". Accepted from publication in *Journal of Geodesy*.

[3] Vallado, D. A. McClain, W. D., "Fundamentals of Astrodynamics and Applications" United State Air Force Academy, 1997.

[4] Montenbruck, O., Eberhard, G., "Satellite Orbits Models, Methods and Applications", 2000.

[5] Schaub H, Junkings J. L., "Analytical Mechanics of Aerospace Systems", 2000.